NAG Fortran Library Routine Document

F08NNF (ZGEEV)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F08NNF (ZGEEV) computes the eigenvalues and, optionally, the left and/or right eigenvectors for an n by n complex nonsymmetric matrix A.

2 Specification

```
      SUBROUTINE F08NNF (JOBVL, JOBVR, N, A, LDA, W, VL, LDVL, VR, LDVR, WORK,
     1                   LWORK, RWORK, INFO)
      INTEGER            N, LDA, LDVL, LDVR, LWORK, INFO
      double precision   RWORK(*)
      complex*16         A(LDA,*), W(*), VL(LDVL,*), VR(LDVR,*), WORK(*)
      CHARACTER*1        JOBVL, JOBVR
```

The routine may be called by its LAPACK name ***zgeev***.

3 Description

The right eigenvector v_j of A satisfies

$$Av_j = \lambda_j v_j$$

where λ_j is the jth eigenvalue of A. The left eigenvector u_j of A satisfies

$${u_j}^H A = \lambda_j {u_j}^H$$

where ${u_j}^H$ denotes the conjugate transpose of u_j.

The matrix B is first reduced to upper Hessenberg form by means of unitary similarity transformations, and the QR algorithm is then used to further reduce the matrix to upper triangular Schur form, T, from which the eigenvalues are computed. Optionally, the eigenvectors of T are also computed and backtransformed to those of A.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: JOBVL – CHARACTER*1 *Input*

On entry: if JOBVL = 'N', the left eigenvectors of A are not computed.

If JOBVL = 'V', the left eigenvectors of A are computed.

Constraint: JOBVL = 'N' or 'V'.

2: JOBVR – CHARACTER*1 *Input*

On entry: if JOBVR = 'N', the right eigenvectors of A are not computed.

If JOBVR = 'V', the right eigenvectors of A are computed.

Constraint: JOBVR = 'N' or 'V'.

3: N – INTEGER *Input*

On entry: n, the order of the matrix A.

Constraint: $\mathrm{N} \geq 0$.

4: A(LDA,$*$) – ***complex*16*** array *Input/Output*

Note: the second dimension of the array A must be at least $\max(1, \mathrm{N})$.

On entry: the n by n matrix A.

On exit: has been overwritten.

5: LDA – INTEGER *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which F08NNF (ZGEEV) is called.

Constraint: $\mathrm{LDA} \geq \max(1, \mathrm{N})$.

6: W($*$) – ***complex*16*** array *Output*

Note: the dimension of the array W must be at least $\max(1, \mathrm{N})$.

On exit: contains the computed eigenvalues.

7: VL(LDVL,$*$) – ***complex*16*** array *Output*

Note: the second dimension of the array VL must be at least $\max(1, \mathrm{N})$.

On exit: if JOBVL = 'V', the left eigenvectors u_j are stored one after another in the columns of VL, in the same order as their corresponding eigenvalues.

If JOBVL = 'N', VL is not referenced. $u_j = \mathrm{VL}(:, j)$, the jth column of VL.

8: LDVL – INTEGER *Input*

On entry: the first dimension of the array VL as declared in the (sub)program from which F08NNF (ZGEEV) is called.

Constraints:

if JOBVL = 'V', $\mathrm{LDVL} \geq \max(1, \mathrm{N})$;
$\mathrm{LDVL} \geq 1$ otherwise.

9: VR(LDVR,$*$) – ***complex*16*** array *Output*

Note: the second dimension of the array VR must be at least $\max(1, \mathrm{N})$.

On exit: if JOBVR = 'V', the right eigenvectors v_j are stored one after another in the columns of VR, in the same order as their corresponding eigenvalues.

If JOBVR = 'N', VR is not referenced.

$v_j = \mathrm{VR}(:, j)$, the jth column of VR.

10: LDVR – INTEGER *Input*

On entry: the first dimension of the array VR as declared in the (sub)program from which F08NNF (ZGEEV) is called.

Constraints:

if JOBVR = 'V', LDVR $\geq \max(1, N)$;
LDVR ≥ 1 otherwise.

11: WORK($*$) – ***complex*16*** array *Workspace*

Note: the dimension of the array WORK must be at least $\max(1, \text{LWORK})$.

On exit: if INFO = 0, WORK(1) returns the optimal LWORK.

12: LWORK – INTEGER *Input*

On entry: the dimension of the array WORK as declared in the (sub)program from which F08NNF (ZGEEV) is called.

If LWORK $= -1$, a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued.

For good performance, LWORK should be generally larger than the minimum, say $\mathrm{N} + nb \times \mathrm{N}$, where nb is the optimum block size for F08NSF (ZGEHRD).

Constraint: LWORK $\geq \max(1, 2 \times \mathrm{N})$.

13: RWORK($*$) – ***double precision*** array *Workspace*

Note: the dimension of the array RWORK must be at least $\max(1, 2 \times \mathrm{N})$.

14: INFO – INTEGER *Output*

On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, the ith argument had an illegal value.

INFO > 0

If INFO $= i$, the QR algorithm failed to compute all the eigenvalues, and no eigenvectors have been computed; elements and $i + 1 : \mathrm{N}$ of W contain eigenvalues which have converged.

7 Accuracy

The computed eigenvalues and eigenvectors are exact for a nearby matrix $(A + E)$, where

$$\|E\|_2 = \mathrm{O}(\epsilon)\|A\|_2,$$

and ϵ is the ***machine precision***. See Section 4.8 of Anderson *et al.* (1999) for further details.

8 Further Comments

Each eigenvector is normalized to have Euclidean norm equal to unity and the element of largest absolute value real and positive.

The total number of floating-point operations is proportional to n^3.

The real analogue of this routine is F08NAF (DGEEV).

9 Example

To find all the eigenvalues and right eigenvectors of the matrix

$$A = \begin{pmatrix} -3.97 - 5.04i & -4.11 + 3.70i & -0.34 + 1.01i & 1.29 - 0.86i \\ 0.34 - 1.50i & 1.52 - 0.43i & 1.88 - 5.38i & 3.36 + 0.65i \\ 3.31 - 3.85i & 2.50 + 3.45i & 0.88 - 1.08i & 0.64 - 1.48i \\ -1.10 + 0.82i & 1.81 - 1.59i & 3.25 + 1.33i & 1.57 - 3.44i \end{pmatrix}.$$

Note that the block size (NB) of 64 assumed in this example is not realistic for such a small problem, but should be suitable for large problems.

9.1 Program Text

Note: the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     F08NNF Example Program Text
*     Mark 21.  NAG Copyright 2004.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NB, NMAX
      PARAMETER        (NB=64,NMAX=10)
      INTEGER          LDA, LDVR, LWORK
      PARAMETER        (LDA=NMAX,LDVR=NMAX,LWORK=(1+NB)*NMAX)
*     .. Local Scalars ..
      INTEGER          I, IFAIL, INFO, J, LWKOPT, N
*     .. Local Arrays ..
      COMPLEX *16      A(LDA,NMAX), DUMMY(1,1), VR(LDVR,NMAX), W(NMAX),
     +                 WORK(LWORK)
      DOUBLE PRECISION RWORK(2*NMAX)
*     .. External Subroutines ..
      EXTERNAL         X04DAF, ZGEEV
*     .. Executable Statements ..
      WRITE (NOUT,*) 'F08NNF Example Program Results'
      WRITE (NOUT,*)
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N
      IF (N.LE.NMAX) THEN
*
*        Read the matrix A from data file
*
         READ (NIN,*) ((A(I,J),J=1,N),I=1,N)
*
*        Compute the eigenvalues and right eigenvectors of A
*
         CALL ZGEEV('No left vectors','Vectors (right)',N,A,LDA,W,DUMMY,
     +              1,VR,LDVR,WORK,LWORK,RWORK,INFO)
         LWKOPT = WORK(1)
*
         IF (INFO.EQ.0) THEN
*
*           Print solution
*
            WRITE (NOUT,*) 'Eigenvalues'
            WRITE (NOUT,99999) (W(J),J=1,N)
*
            WRITE (NOUT,*)
            IFAIL = 0
            CALL X04DAF('General',' ',N,N,VR,LDVR,'Eigenvectors',IFAIL)
         ELSE
            WRITE (NOUT,*)
            WRITE (NOUT,99998) 'Failure in ZGEEV.  INFO = ', INFO
         END IF
*
*        Print workspace information
```

```
*
         IF (LWORK.LT.LWKOPT) THEN
            WRITE (NOUT,*)
            WRITE (NOUT,99997) 'Optimum workspace required = ', LWKOPT,
     +        'Workspace provided         = ', LWORK
         END IF
      ELSE
         WRITE (NOUT,*) 'NMAX too small'
      END IF
      STOP
*
99999 FORMAT ((3X,4(' (',F7.4,',',F7.4,')',:)))
99998 FORMAT (1X,A,I4)
99997 FORMAT (1X,A,I5,/1X,A,I5)
      END
```

9.2 Program Data

```
F08NNF Example Program Data

  4                                                            :Value of N

(-3.97, -5.04)  (-4.11,  3.70)  (-0.34,  1.01)  ( 1.29, -0.86)
( 0.34, -1.50)  ( 1.52, -0.43)  ( 1.88, -5.38)  ( 3.36,  0.65)
( 3.31, -3.85)  ( 2.50,  3.45)  ( 0.88, -1.08)  ( 0.64, -1.48)
(-1.10,  0.82)  ( 1.81, -1.59)  ( 3.25,  1.33)  ( 1.57, -3.44) :End of matrix A
```

9.3 Program Results

```
 F08NNF Example Program Results

 Eigenvalues
    (-6.0004,-6.9998) (-5.0000, 2.0060) ( 7.9982,-0.9964) ( 3.0023,-3.9998)

 Eigenvectors
          1       2       3       4
 1   0.8457 -0.3865 -0.1730 -0.0356
     0.0000  0.1732  0.2669 -0.1782

 2  -0.0177 -0.3539  0.6924  0.1264
     0.3036  0.4529  0.0000  0.2666

 3   0.0875  0.6124  0.3324  0.0129
     0.3115  0.0000  0.4960 -0.2966

 4  -0.0561 -0.0859  0.2504  0.8898
    -0.2906 -0.3284 -0.0147  0.0000
```